

Phonon-particle coupling effects in odd-even double mass differences of semi-magic nuclei

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Submitted November 29, 2016

A method is developed to consider the particle-phonon coupling (PC) effects in the calculation of the odd-even double mass differences (DMD) in semi-magic nuclei starting from the free NN -potential. The PC correction $\delta\Sigma^{\text{PC}}$ to the mass operator Σ is found in g_L^2 -approximation, g_L being the vertex of creating the L -phonon. The tadpole term of the operator $\delta\Sigma^{\text{PC}}$ is taken into account. The method is based on a direct, without any use of the perturbation theory, solution of the Dyson equation with the mass operator $\Sigma(\varepsilon)=\Sigma_0+\delta\Sigma^{\text{PC}}(\varepsilon)$ for finding the single-particle energies and Z -factors. In its turn, they are used as an input for finding different PC corrections to the DMD values. Results for a chain of even semi-magic nuclei $^{200-206}\text{Pb}$ show that the inclusion of the PC corrections makes agreement with the experimental data significantly better.

PACS: 21.60.-n; 21.65.+f; 26.60.+c; 97.60.Jd

Recently, we developed a method to find the particle-phonon coupling (PC) corrections to the odd-even double mass differences (DMD) of magic nuclei starting from the free NN -potential (FP) [1, 2]. This problem is closely related to that of finding the pairing gap Δ in nuclei in terms of the FP, which was rather popular in the last decade, see the review article [3] and Refs. therein. The approach was essentially based on the so-called g_L^2 -approximation, where g_L is the vertex of the L -phonon creation, which works well in magic nuclei. The use of the perturbation theory (PT) in the PC correction $\delta\Sigma^{\text{PC}}(\varepsilon)$ with respect to the initial mass operator Σ_0 for finding the single-particle (SP) energies is an additional approximation used in [1, 2]. In semi-magic nuclei, the latter is regularly not valid due to presence of the low-lying collective 2^+ phonon. In its turn, it results in the appearance of small energy denominators in the expression for the operator $\delta\Sigma^{\text{PC}}(\varepsilon)$ and in the one for the phonon induced interaction \mathcal{V}_{ind} as well, making the use of the PT inapplicable. Recently the problem for SP energies was resolved by us in [4] by direct solving the Dyson equation with the mass operator $\Sigma(\varepsilon)=\Sigma_0+\delta\Sigma^{\text{PC}}(\varepsilon)$ without any use of the PT. In this work, we continue development of this method to find the DMD values for semi-magic nuclei.

As in [4], we deal with a normal sub-system of the nucleus under consideration, therefore the general set of equations for DMD in magic nuclei [1, 2] remains

valid. However, the method of their solution for semi-magic nuclei changes significantly. We limit ourselves with Pb isotopes, thus, the proton DMD values will be considered only which are defined in terms of the nuclear masses $M(N, Z)$, where N and Z are neutron and proton numbers correspondingly in the nucleus under consideration, as follows:

$$D_{2p}^+(N, Z)=M(N, Z+2)+M(N, Z)-2M(N, Z+1), \quad (1)$$

$$D_{2p}^-(N, Z)=-M(N, Z-2)-M(N, Z)+2M(N, Z-1). \quad (2)$$

To make the discussion more transparent, we repeat schematically the main relations for DMD of semi-magic nuclei without PC corrections [5, 6]. Let us start from the Lehmann expansion for the two-particle Green function K . In the SP wave functions $|1\rangle=|n_1, l_1, j_1, m_1\rangle$ representation, it reads [7]:

$$K_{12}^{34}(E)=\sum_s \frac{\chi_{12}^s \chi_{34}^{s+}}{E-E_s^{+, -} \pm i\gamma}, \quad (3)$$

where E is the total energy in the two-particle channel and $E_s^{+, -}$ denote the energies of eigenstates of nuclei with two particles or two holes, respectively, added to the original nucleus. The lowest ones of them determine the mass differences entering Eqs. (1) and (2).

The Green function K relates to the two-particle interaction amplitude Γ as follows:

$$K = K_0 + K_0 \Gamma K_0, \quad (4)$$

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where $K_0 = GG$, G being the one-particle Green function. Within the Brueckner theory, the amplitude Γ obeys the Bethe–Goldstone equation:

$$\Gamma = \mathcal{V} + \mathcal{V}GG\Gamma, \quad (5)$$

where \mathcal{V} is the FP.

This equation is in many ways similar to the Brueckner theory gap equation [3] possessing the same problem of slow convergence. For the Argonne v_{18} potential we use as the FP, SP states with energies up to 1 GeV should be included into the SP space to obtain a good accuracy. Therefore, the same two-step renormalization method was used in [5, 6] which was developed previously for the pairing problem in Refs. [8, 9]. The complete two-particle Hilbert space S of the problem is split in the model subspace S_0 , including the SP states with energies less than a separation energy E_0 , and the complementary one, S' . In practice, in all the articles cited above we use $E_0=40$ MeV. In the result, Eq. (5) is split into two ones: in the model space,

$$\Gamma = \mathcal{V}_{\text{eff}} + \mathcal{V}_{\text{eff}}GG\Gamma|_{S_0}, \quad (6)$$

and in the subsidiary space:

$$\mathcal{V}_{\text{eff}} = \mathcal{V} + \mathcal{V}GG\mathcal{V}_{\text{eff}}|_{S'}. \quad (7)$$

To solve the last equation for the effective interaction (EI) \mathcal{V}_{eff} , a method of “Local Potential Approximation” was developed for the pairing problem [8, 9], with the use of plane waves instead of the exact SP states $|\lambda\rangle$. In the DMD problem, [5, 6], it turned out to be also applicable. As to the first of these two equations, it was solved in the space S_0 directly, without additional approximations. In this case, it is convenient to carry out the integration in Eq. (6) of the product GG of two Green functions over the relative energy:

$$A_{12}(E) = \int \frac{d\varepsilon}{2\pi i} G_1\left(\frac{E}{2} + \varepsilon\right) G_2\left(\frac{E}{2} - \varepsilon\right) = \frac{1 - n_1 - n_2}{E - \varepsilon_1 - \varepsilon_2}, \quad (8)$$

where $\varepsilon_{1,2}$ are the SP energies and $n_{1,2}=(0;1)$, the corresponding occupation numbers.

In Refs. [8, 9], the “semi-microscopic model” was suggested to take into account approximately many-body theory corrections to the EI (7) found in terms of the FP. The main term (7) is supplemented with a phenomenological δ -function addendum:

$$\mathcal{V}_{\text{eff}}(\mathbf{r}_1, \dots, \mathbf{r}_4) = V_{\text{eff}}^{\text{free}}(\mathbf{r}_1, \dots, \mathbf{r}_4) + \gamma C_0 \frac{\rho(r_1)}{\bar{\rho}(0)} \prod_{k=2}^4 \delta(\mathbf{r}_1 - \mathbf{r}_k). \quad (9)$$

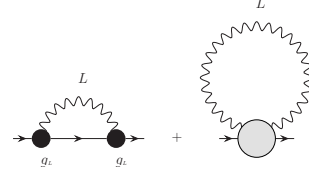


Fig. 1. PC corrections to the mass operator. The black circle is the vertex g_L of creating the L -phonon, the gray blob denotes the phonon “tadpole” term.

Here $\rho(r)$ is the density of nucleons of the kind under consideration (protons in our case), $C_0=300$ MeV·fm³ is the usual normalization factor of the theory of finite Fermi systems [7], and γ is a dimensionless phenomenological parameter. The quantity $\bar{\rho}(0)$ in the denominator is the average central density. The value of $\gamma=0.06$ was found in the references above as an optimal one for describing the bulk of data on the pairing gap. It turned out to be successful also for describing the DMD values in magic and semi-magic nuclei [5, 6] without PC corrections. In Refs. [1, 2], it was shown that, after inclusion of the PC corrections in magic nuclei, this addendum is diminished to $\gamma=0 \div 0.03$. Here, a similar analysis is carried out for semi-magic nuclei.

The DMD values without PC corrections are determined with the eigenenergies E_s of the following equation [5]:

$$(E_s - \varepsilon_1 - \varepsilon_2) \chi_{12}^s = (1 - n_1 - n_2) \sum_{34 \in S_0} (\mathcal{V}_{\text{eff}})_{12}^{34} \chi_{34}^s. \quad (10)$$

It is different from the Shrödinger equation for two interacting particles in an external potential well only by the factor $(1 - n_1 - n_2)$ reflecting the Pauli principle. Just as in the pairing problem, the angular momenta of two-particle states $|12\rangle$, $|34\rangle$ are coupled to the total angular momentum $I=0$ ($S=0, L=0$).

To include the low-lying phonons, we should take into account that they influence mainly the SP states close to the Fermi level. Therefore, it is reasonable to make an additional renormalization of Eq. (10) by splitting our model space S_0 to the “valence” subspace S_0^v , containing two shells adjacent to the Fermi level, and the subsidiary part S_0' of the model space.

To find the SP energies in the space S_0^v with account for the PC effects, we solve the following equation:

$$(\varepsilon - H_0 - \delta\Sigma^{\text{PC}}(\varepsilon)) \phi = 0, \quad (11)$$

where H_0 is the quasiparticle Hamiltonian with the spectrum ε_λ , and $\delta\Sigma^{\text{PC}}$ is the PC correction to the quasiparticle mass operator. All diagrams for it of the order g_L^2 are displayed in Fig. 1. If the PT in $\delta\Sigma^{\text{PC}}$ with

respect to H_0 is valid, as it occurs in magic nuclei, we have for the PC corrected SP energies [1, 2, 10]:

$$\begin{aligned} \varepsilon_\lambda &= \varepsilon_\lambda + Z_\lambda \delta \Sigma_{\lambda\lambda}^{\text{PC}}(\varepsilon_\lambda), \\ Z_\lambda &= \left(1 - \left(\frac{\partial}{\partial \varepsilon} \delta \Sigma^{\text{PC}}(\varepsilon) \right)_{\varepsilon=\varepsilon_\lambda} \right)^{-1}. \end{aligned} \quad (12)$$

In this case, each SP state in the valent space generates the single PC corrected one: $|\lambda\rangle \rightarrow |\tilde{\lambda}\rangle = \sqrt{Z_\lambda}|\lambda\rangle$, and we obtain from (10) the PC corrected equation in the valence space:

$$(E_s - \tilde{\varepsilon}_1 - \tilde{\varepsilon}_2) \chi_{12}^s = (1 - n_1 - n_2) \sum_{34 \in S_0^v} (\tilde{\mathcal{V}}_{\text{eff}})_{12}^{34} \chi_{34}^s, \quad (13)$$

$$\langle 11' | \tilde{\mathcal{V}}_{\text{eff}} | 22' \rangle = \sqrt{Z_1 Z_{1'} Z_2 Z_{2'}} \langle 11' | \mathcal{V}_{\text{eff}}^v + \mathcal{V}_{\text{ind}} | 22' \rangle, \quad (14)$$

where the EI $\mathcal{V}_{\text{eff}}^v$ obeys the equation similar to (5), but in the subsidiary model space S_0^v . This equation is solved directly, without any approximation. The phonon induced interaction \mathcal{V}_{ind} is displayed in Fig. 2. The explicit expression of the matrix element of \mathcal{V}_{ind} is as follows [1, 2]:

$$\begin{aligned} \langle 11' | \mathcal{V}_{\text{ind}} | 22' \rangle &= - \frac{2\omega_L}{\sqrt{(2j_1+1)(2j_2+1)}} \\ &\times \frac{(\langle j_1 l_1 | Y_L | j_1 l_1 \rangle (g_L)_{11'}) (\langle j_2 l_2 | Y_L | j_2 l_2 \rangle (g_L)_{22'})^*}{\omega_L^2 - (\varepsilon_2 - \varepsilon_1)^2}, \end{aligned} \quad (15)$$

where ω_L is the excitation energy of the L -phonon, $\langle ||Y_L|| \rangle$ stands for the reduced matrix element, and $(g_L)_{ii'}$ are the radial matrix elements of the vertex $g_L(r)$. Notice that we deal with the channel with $I=0, S=0, L=0$. Therefore, the states i, i' in (14) possess the same SP angular momenta, $j_1=j_{1'}, l_1=l_{1'}; j_2=j_{2'}, l_2=l_{2'}$.

In the valence subspace we consider, always there is only one state for each (l, j) value. Therefore, we need only diagonal elements $\delta \Sigma_{\lambda\lambda}$. Explicit expression for the corresponding pole term is as follows [1, 10]:

$$\begin{aligned} \delta \Sigma_{\lambda\lambda}^{\text{pole}}(\varepsilon) &= \\ &= \sum_{\lambda' M} |\langle \lambda' | g_{LM} | \lambda \rangle|^2 \left(\frac{n_{\lambda'}}{\varepsilon + \omega_L - \varepsilon_{\lambda'}} + \frac{1 - n_{\lambda'}}{\varepsilon - \omega_L - \varepsilon_{\lambda'}} \right). \end{aligned} \quad (16)$$

In [1, 2], the above equations were successfully applied to finding PC corrections to DMD values in magic nuclei. In semi-magic nuclei we deal, the PT solution (12) becomes regularly not valid because of the presence of the low-lying 2^+ state. For example, in the ^{204}Pb nucleus we have $\omega_2=0.88$ MeV and $\varepsilon(3s_{1/2}) - \varepsilon(2d_{3/2})=0.79$ MeV. As a result, catastrophically small energy denominators of $\simeq 0.1$ MeV appear

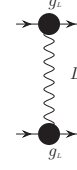


Fig. 2. The L -phonon induced interaction.

in Eq. (15) and in Eq. (16) at $\varepsilon=\varepsilon_\lambda$. To find a direct solution of Eq. (11) in the valence space, instead of the PT one (12), is a key step to solve the problem. It was made by us recently [4]. Here, we describe in short this method.

As it was mentioned above, only diagonal matrix elements of $\delta \Sigma_{\lambda\lambda}$ participate in equations in the valence subspace. In the result, Eq. (11) reduces as follows:

$$\varepsilon - \varepsilon_\lambda - \delta \Sigma_{\lambda\lambda}^{\text{PC}}(\varepsilon) = 0, \quad (17)$$

where $\delta \Sigma^{\text{PC}}(\varepsilon) = \delta \Sigma^{\text{pole}}(\varepsilon) + \delta \Sigma^{\text{tad}}$, with obvious notation. The tadpole term does not depend on the energy, therefore the singular points of Eq. (17) coincide with poles of Eq. (16). They can be readily found from (16) in terms of ε_λ and ω_L . It can be easily seen that the lhs of Eq. (17) always changes sign between any couple of neighboring poles, therefore the corresponding solution ε_λ^i can be found with usual methods. In this notation, λ is just the index for the initial SP state from which the state $|\lambda, i\rangle$ originated. The corresponding SP strength distribution factors (S -factors) are now determined with the energy derivative in the exact SP energy value:

$$S_\lambda^i = \left(1 - \left(\frac{\partial}{\partial \varepsilon} \delta \Sigma^{\text{PC}}(\varepsilon) \right)_{\varepsilon=\varepsilon_\lambda^i} \right)^{-1}. \quad (18)$$

In the result, the one-particle Green function in the subspace S_0^v , which, without PC corrections, for each λ contained only one pole, $G_\lambda(\varepsilon) = (\varepsilon - \varepsilon_\lambda \pm i\delta)^{-1}$, is now split into a sum of poles:

$$\tilde{G}_\lambda(\varepsilon) = \sum_i G_\lambda^i(\varepsilon), \quad G_\lambda^i(\varepsilon) = \frac{S_\lambda^i}{\varepsilon - \varepsilon_\lambda^i \pm i\delta}. \quad (19)$$

The correct scheme should involve, for the valence subspace, the insertion of the partial Green functions G_λ^i instead of G_λ . As a result, a total number of the above relations strongly grows going from $\varepsilon_\lambda, Z_\lambda$ to $\varepsilon_\lambda^i, S_\lambda^i$. Such approach is technically possible but it is rather cumbersome. We prefer to use an approximate approach suggested in [4] for finding the PC corrected SP energies. As the analysis shows, there are two different kinds of solutions of (17). Their examples are shown in Table 1. There are “good” SP states with a dominating

Table 1. Examples of solutions of Eq. (17) for protons in ^{204}Pb .

λ	i	ε_{λ}^i (MeV)	S_{λ}^i
$2d_{5/2}$	1	-11.817	0.314×10^{-2}
	2	-11.150	0.139
	3	-9.799	0.516×10^{-1}
	4	-8.580	0.312
	5	-8.195	0.295
	6	-7.404	0.171
$3s_{1/2}$	1	-9.877	0.608×10^{-1}
	2	-8.536	0.604×10^{-1}
	3	-6.493	0.839

term $|\lambda, i_0\rangle$ for which the following prescription similar to (12) looks natural:

$$\tilde{\varepsilon}_{\lambda} = \varepsilon_{\lambda}^{i_0}; \quad Z_{\lambda} = S_{\lambda}^{i_0}. \quad (20)$$

It should be stressed that these relations remind the PT solution (17) only in the form. Indeed, now $\varepsilon_{\lambda}^{i_0}$ is one of the exact solutions of (17). In addition, the Z -factor is determined now with the energy derivative (18) of the mass operator in this exact SP energy value.

There are also the cases of a strong spread where several terms $|\lambda, i\rangle$ possess comparable strengths S_{λ}^i . In such cases, the following generalization of Eq. (20) was suggested in [4]:

$$\tilde{\varepsilon}_{\lambda} = \frac{1}{Z_{\lambda}} \sum_i S_{\lambda}^i \varepsilon_{\lambda}^i, \quad Z_{\lambda} = \sum_i S_{\lambda}^i. \quad (21)$$

According to [4], all the states $|\lambda, i\rangle$ with comparably large strengths should be included into both the above sums.

In this approximate scheme, the exact Green function (19) is changed with the approximate one,

$$\tilde{G}_{\lambda}(\varepsilon) = \frac{Z_{\lambda}}{\varepsilon - \tilde{\varepsilon}_{\lambda} \pm i\delta}, \quad (22)$$

where $\tilde{\varepsilon}_{\lambda}$ and Z_{λ} are taken from (20) or (21), depending on the type of the solution we deal.

The final recipe to find DMD values in semi-magic nuclei we suggest is to use these non-perturbative SP energies and Z -factors from Eqs. (20) or (21) in the set of equations (13)–(15), instead of the PT values used in [1, 2] for magic nuclei. In this work, we test this method considering the same four even semi-magic $^{200-206}\text{Pb}$ isotopes, as in [4] where the method of direct solution of Eq. (17) without any PT was developed. Other technical details are also the same as in [4], i.e. we use the

Table 2. PC corrected proton single-particle characteristics $\tilde{\varepsilon}_{\lambda}$ (MeV) and Z_{λ} of even Pb isotopes.

Nucleus	λ	ε_{λ}	$\tilde{\varepsilon}_{\lambda}$	Z_{λ}
^{200}Pb	$1i_{13/2}$	-0.26	-0.93	0.64
	$2f_{7/2}$	-1.05	-1.35	0.83
	$1h_{9/2}$	-2.33	-2.65	0.67
	$3s_{1/2}$	-5.81	-5.32	0.77
	$2d_{3/2}$	-6.67	-5.88	0.52
	$1h_{11/2}$	-7.06	-6.39	0.72
	$2d_{5/2}$	-7.88	-7.60	0.88
	$1g_{7/2}$	-9.97	-9.89	0.91
^{202}Pb	$1i_{13/2}$	-0.74	-1.40	0.65
	$2f_{7/2}$	-1.52	-1.81	0.83
	$1h_{9/2}$	-2.86	-3.16	0.68
	$3s_{1/2}$	-6.26	-5.75	0.77
	$2d_{3/2}$	-7.09	-6.31	0.54
	$1h_{11/2}$	-7.52	-6.87	0.73
	$2d_{5/2}$	-8.34	-8.04	0.87
	$1g_{7/2}$	-10.46	-10.38	0.91
^{204}Pb	$1i_{13/2}$	-1.21	-1.63	0.71
	$2f_{7/2}$	-2.01	-2.24	0.87
	$1h_{9/2}$	-3.36	-3.45	0.76
	$3s_{1/2}$	-6.72	-6.49	0.84
	$2d_{3/2}$	-7.51	-7.03	0.58
	$1h_{11/2}$	-7.98	-7.51	0.81
	$2d_{5/2}$	-8.80	-8.63	0.92
	$1g_{7/2}$	-10.93	-10.49	0.54
^{206}Pb	$1i_{13/2}$	-1.67	-1.94	0.77
	$2f_{7/2}$	-2.51	-2.81	0.82
	$1h_{9/2}$	-3.82	-3.77	0.84
	$3s_{1/2}$	-7.18	-7.10	0.89
	$2d_{3/2}$	-7.91	-7.64	0.65
	$1h_{11/2}$	-8.42	-8.05	0.88
	$2d_{5/2}$	-9.28	-9.16	0.95
	$1g_{7/2}$	-11.36	-11.12	0.90

DF3-a version [11] of the Fayans energy density functional [12] to generate the self-consistent basis $|\lambda\rangle$ and take into account two L -phonons, 2_1^+ and 3_1^- . Their characteristics may be found in [4].

Table 2 contains the values of characteristics of the approximate PC corrected Green function (22) we use. Some of them are different of those in the corresponding table in [4]. The reason of that is in different ways

Table 3. Double mass differences D_2 (MeV) with and without account for PC effects and separate PC corrections to the D_2 values in even Pb isotopes.

A		$D_2^{(0)}$	$D_2(\gamma=0.06)$	$\delta D_2(Z)$	$\delta D_2(\mathcal{V}_{\text{ind}})$	$\delta D_2(\delta\varepsilon)$	δD_2^{PC}	D_2^{PC}	$D_2^{\text{PC}}(\gamma=0.03)$	D_2^{exp}
200	D_2^-	1.448	0.923	-0.897	1.807	0.892	-0.208	1.240	1.166	1.0878(675)
	D_2^+	-2.099	-1.381	1.482	-1.603	-0.899	0.767	-1.332	-1.219	-1.345(56)
202	D_2^-	1.492	0.936	-0.937	1.521	0.760	-0.303	1.189	1.117	1.0856(324)
	D_2^+	-2.139	-1.396	1.500	-1.647	-0.791	0.801	-1.338	-1.225	-1.233(41)
204	D_2^-	1.549	0.956	-0.875	1.311	0.886	-0.630	0.919	0.824	0.9333(44)
	D_2^+	-2.185	-1.415	1.373	-1.287	0.780	0.939	-1.246	-1.110	-1.1694(152)
206	D_2^-	1.617	0.981	-0.794	0.290	0.510	-0.613	1.004	0.856	0.8347(43)
	D_2^+	-2.236	-1.439	-1.119	-0.189	-0.369	-0.918	-1.318	-1.136	-1.1461(78)
208	D_2^-	1.680	1.000	-0.824	-0.083	0.569	-0.745	0.935	0.915	0.6271(312)
	D_2^+	-2.286	-1.466	1.049	-0.167	-0.329	0.830	-1.456	-1.276	-1.1844(52)
$\langle \delta D_2 \rangle_{\text{rms}}$		0.849	0.216	-	-	-	-	0.167	0.087	-

to choose the components 'i' in the sums of (21) for solutions with large spread. In [4], we oriented to a procedure which is used for finding the experimental SP energies when, in an odd nucleus under consideration, the excitations with the same j^π are included in the sums of (21) provided they possess comparatively large spectroscopic factors $S^i(j^\pi)$. This recipe is reasonable for theoretical applications provided the exact Green function (19) is integrated with a smooth energy function. Now, this is not the case. Indeed, the use of two Green functions (19) to find an exact expression for the induced interaction instead of (15) will result in a similar expression with the denominators $(\omega_L^2 - (\varepsilon_2^{i_2} - \varepsilon_1^{i_1})^2)$, with obvious notation. In the case of the 2^+ phonon, $\omega_2 \simeq 1$ MeV, this is rather sharp function of two energies in this expression, and contributions of the terms with smaller denominators are enhanced. Therefore, in choosing the terms 'i' in Eq. (21) now we take into account the "denominator factor", in addition to the value of the spectroscopic factor.

Table 3 contains the results of the calculations of the DMD values with and without account for PC effects. The initial DMD value denoted as $D_2^{(0)}$ is found on the base of the FP, i.e. from Eqs. (9) and (10) at $\gamma=0$. The next column contains similar quantity found at $\gamma=0.06$, which is the optimal value of this parameter found without account for PC effects [3, 5, 6]. The next three columns present separate contributions of three different PC effects under two others being switched off. For example, $\delta D_2(Z)$ is the difference between the D_2 value, found from Eqs. (13) and (14) at $\mathcal{V}_{\text{ind}}=0$ and $\tilde{\varepsilon}_\lambda=\varepsilon_\lambda$, and the initial value of $D_2^{(0)}$. The next difference $\delta D_2(\mathcal{V}_{\text{ind}})$

is found according the same scheme, but now the induced interaction \mathcal{V}_{ind} in (14) is taken into account at $Z_1=Z_2=1$. Finally, the quantity $\delta D_2(\delta\varepsilon)$ is found from (13) when the difference of the SP energies $\tilde{\varepsilon}_\lambda$ from the initial values ε_λ is taken into account only. The quantity $\delta D_2^{\text{PC}}=D_2^{\text{PC}}-D_2^{(0)}$ shows the total PC effect. It should be stressed that the total PC correction does not equal to the sum of the three separate ones as there is some interference. For example, the induced interaction in (14) is multiplied with the Z -factors. For completeness, we added the PT results for the magic ^{208}Pb from [1, 2]. As we see from the table, the corrections due to the Z -factor and due to the induced interaction are, as a rule, very big and have opposite signs, the result being essentially smaller in absolute value of each of them. Sometimes, the SP energy correction is also significant. The last two columns preceding the experimental one contain the total PC corrected DMD values. The second of them includes also the phenomenological addendum in Eq. (9) with $\gamma=0.03$. This value is two times less than the optimal one found previously without PC corrections. We see, that both the PC corrected results for DMDs agree with experiment sufficiently well, especially the last of them. To estimate the agreement with experimental data quantitatively, the rms differences between theoretical predictions and data are given in the end of Table 3 for four versions of the theory: the pure FP calculation, the result of the semi-microscopic model (9) with the value of $\gamma=0.06$ found previously in calculations without PC corrections, and two results with the PC corrections, with $\gamma=0$ and $\gamma=0.03$. The rms values of this differences are given in the last line of Table 3.

One can see that inclusion of the PC corrections makes agreement with experiment essentially better, especially in a combination with a small phenomenological addendum of the semi-microscopic model [8, 9] with $\gamma=0.03$.

To conclude, we developed for semi-magic nuclei a method of finding the PC corrections to the DMD values in the approach starting from a free NN potential. The main difference from the similar problem for magic nuclei [1, 2] is that the PT used in magic nuclei for finding SP energies and Z -factors is now unapplicable. Instead of this, we apply the method of the direct solution of the Dyson equation, without any use of PT, developed by us recently [4]. The SP energies and Z -factors, found in such a way, are now used in all expressions for the PC corrections under consideration. Account for the PC corrections makes agreement of the DMD values with experiment significantly better, especially in the version of the semi-microscopic model with the value of the phenomenological parameter $\gamma=0.03$, which is two times less than the one in the approach without PC corrections. As it was discussed when the semi-microscopic model was suggested [3, 8, 9], the phenomenological addendum proportional to the parameter γ should take into account approximately three many-body effects changing the result of a simple FP calculation. These are the difference of the effective mass of a nucleon inside a nucleus from a bare one, the contribution from high-lying nuclear excitations as Giant Resonances, and finally, the PC effects. However it is known [3], that the first two effects possess opposite signs and cancel each other significantly. In such a situation, the PC correction takes center stage. Our calculation confirms this analysis. Indeed, the account for PC corrections diminishes the value of γ , as a minimum, in two times. The analysis of a more wide base of data is necessary for a more accurate estimate of the γ value. In addition, the next refining of the calculation scheme is desirable which includes the change of the approximate single-particle Green functions we use by the exact representation of Eq. (19), where each single-particle pole is split into a sum of several poles.

This research was supported by the Russian Science Foundation, Grants Nos. 16-12-10155 and 16-12-10161. E.S. and S.P. thank the INFN, Sezione di Catania, for hospitality. Calculations were partially carried out on the Computer Center of Kurchatov Institute.

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